A study of various fractional integral transforms with their Applications

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Abstract:

we discuss standard approaches to the problem of fractional integral transforms involving conformable derivative, namely the Laplace transforms of fractional derivatives, Fourier transforms of fractional derivative and Mellin transforms of fractional derivative. We prove the basic properties of fractional transforms derivatives including the rules for their compositions and the conditions for the equivalence of various definitions. Further, we give a brief survey of the basic methods for the solving of linear fractional transforms derivatives and mention limits of their usability. In particular, we formulate the theorem describing the structure of the initial-value problem for linear two-term equations. Finally, we consider introduced the new fractional integral transform and proved some of its basic properties. Then obtained new fractional integral transform of some elementary functions and conformable derivative. Further, we define new convolution operation, proved convolution theorem for new fractional integral transform and commutative semi group property of new convolution operation. At last the new fractional integral transform is applied to obtain the solutions of diffusion equation and heat-transfer problem involving conformable derivative.

Introduction:

The subject of fractional transform derivative caught the attention of other great mathematicians, many of whom directly or indirectly contributed to its development. They included Euler, Laplace, Fourier, Lacroix, Abel, Riemann and Liouville. Ravi P. Agarwal, Mouffak Benchohra, And Samira Hamani.(2009). The concept of integral transforms is originated from the Fourier integral formula. The importance of integral transforms is that they provide powerful operational method for solving initial and boundary value problems. The operational calculus of integral transform is used to solve the differential, fractional differential and integral equations arising in applied mathematics, mathematical physics and Engineering science. In recent years, fractional order differential equations have become an important tool in mathematical modeling . The idea of fractional is natural $\partial u/\ \partial x$ and $(\partial^2 u)/(\partial x^2)$ exist, $(\partial^{1.5} u)/(\partial x^{1.5})$

maybe exists too. Fractional equations can be used to describe some physical phenomena more accurately than the classical integer order differential equation . Fractional differential equations (FDEs) provide a powerful instrument for the description of memory and hereditary properties of different substances. The fractional diffusion equations play an important role in dynamical systems of semi-conductor research, hydrogeology, bioinformatics and finance.

Definition of the Laplace transforms of fractional derivative:

The Laplace transform of the function $F(s)$ of the complex variable s defined as

$$\boldsymbol{F(s) = L\{f(t); s\} = \int_0^\infty e^{-st}\ f(t)dt}$$

Definition of Laplace transforms of the Riemann Liouville Fractional Derivative :

The Laplace transforms of the Riemann Liouville fractional integral of order $p > 0$ defined as

$$\boldsymbol{L\{D_t^{-p} f(t); s\} = L\{D_t^{-p} f(t); s\} = s^{-p} F(s)}$$

Definition of the Mellin transforms of fractional derivative :

The Mellin integral transform F(s) of a function f (t). which is defined in the interval $(0, \infty)$ is

$$\boldsymbol{F(s) = M\{f(t); s\} = \int_0^\infty f(t)\, t^{s-1} dt}$$

Objectives Of The Research Work:

The synopsis will be divided into different chapters such as introduction of the work, solution of various fractional integral transforms and transforms derivatives, result and discussion. further, each chapter will be subdivided into different sections if required .

Review of literature and development in the subject area:

1)Brief history:

Integral transformations have been successfully used for almost two centuries in solving many problems in applied mathematics, mathematical physics, and engineering science. Historically, the origin of the integral transforms including the Laplace and Fourier transforms can be traced back to celebrated work of P. S. Laplace (1749–1827) on probability theory in the 1780s and to monumental treatise of Joseph Fourier (1768–1830) on La Th´eorie Analytique de la Chaleur published in 1822. In fact, Laplace's classic book on La Th´eorie Analytique des Probabilities included some basic results of the Laplace transform which was one of the oldest and most commonly used integral transforms available in the mathematical literature. This has effectively been used in finding the solution of linear differential equations and integral equations. On the other hand, Fourier's treatise provided the modern mathematical theory of heat conduction, Fourier series, and Fourier integrals with applications. In his treatise, Fourier stated a remarkable result that is universally known as the Fourier Integral Theorem.

2)Recent Works:

On the other side, many of researchers considers fractional integral transform Pravinkumar V. Dole and S.K.Panchal, A. Kashuri, A. Fundo, R. Liko,

they development a new integral transform is derived from the classical Fourier integral. A new integral transform was introduced by PhD student Artion Kashuri and Associate Professor Akli Fundo to facilitate the process of solving fractional differential equations in the time domain. Typically, Fourier, Laplace, Sumudu and Elzaki transforms are the convenient mathematical tools for solving differential equations. A new integral transform is defined for functions of exponential order. This transform has deeper connection with the Laplace transform.

Objective of research:

This purpose of this study is to show the applicability of this interesting new transform and its efficiency in solving fractional differential equations. New integral transform is a powerful tool and fullness of these lies in their ability to transform differential equations into algebraic equations which allows simple and systematic solution procedures. In the past two decades, the widely investigated subject of fractional calculus has remarkably gained importance and popularity due to its demonstrated applications in numerous diverse fields of science and engineering. These contributions to the fields of science and engineering are based on the mathematical analysis. It covers the widely known classical fields such as Abel's integral equation and viscoelasticity We prove well-posedness of a general class of fractional-order elliptic problems.

Mothodology to be adopted:

The algorithms will be investigated. The solution of each problems that we set up in the introduction will proved or disproved with such counter examples.

Importance:

Applied mathematics has long provided a rich source of motivation for developments in Fractional integral transforms Also, the fractional transforms derivatives plays a central role in linear systems and control theory and dynamical systems.

Proposed work plan and applications:

1)Interdisplenary of application:

A number of intriguing questions remain unanswered about fractional transforms derivatives.

2)Plan of research:

More than four scientific papers are expected to be published in international journals. The results will be presented at least at two international conferences with few dedicated workshops, and networking environment and opportunities to share the scientific results with the European community. Participation to scientific workshops, meetings, specialized training and short-term research visits are expected during the PhD project.

References:

1) Pravinkumar V. Dole and S.K.Panchal, On a New Fractional Integral Transform and its Applications, Annals of Pure and Applied Mathematics Vol. 14, No. 2, (2017) 321-326

2) S.C.Sharma and R.K.Bairwa, A reliable treatment of iterative laplace transform method for fractional telegraph equations, Annals of Pure and Applied Mathematics*, 9(1) (2015) 81-89.*

3) K.B.Oldham and J.Spanier, The Fractional Calculus*, Academic Press, New York, NY, USA, (1974).*

4) L.Debnath, Recent applications of fractional calculus to science and engineering, International Journal of Mathematics and Mathematical Sciences*, 54 (2003) 3413-3442.*

5) Rudolf Hilfer , Yury Luchko , ·Zivorad Tomovski "operational method for the solution of fractional differential equations with generalized Riemann liouville fractional derivatives"Diogenes Co. (Bulgaria). 2009.

6) Igor Podlubny," Fractional Differential Eqnations," San Diego: Academic Press . 1999 .

7) Adam Loverro " Fractional Calculus: History, Definitions and Applications for the Engineer " Department of Aerospace and Mechanical Engineering University of Notre Dame .2004.

8) A.A. Kilbas, H. M. Srivastava,And J.J. Trujillo" Theory And Applications Of Fractional Differential equations" Amsterdam:Elsevier. 2006.

9) Dr. Dimple Singh."Integral Transform of Fractional Derivatives "International Journal of Recent Research Aspects ISSN: 2349-7688, Vol. 4, Issue 1, March 2017, pp. 74-76

10) L. Debnath, Integral Transforms and Their Applications, CRC Press, Florida,1995.

11) K. S. Miller, "An Introduction to the Fractional Calculus and Fractional Differential Equations", Wiley Interscience; (1993).1 edition, ISBN-13: 978-0471588849

12) I.N.Sneddon, The Use of Integral Transforms, *McGraw-Hill International Editions, New York, (1972).*

13) Dzherbashyan, M.M. (1966). Integral Transforms and Representation of Functions in Complex Domain (in Russian), Nauka, Moscow.

14) A. Kashuri, A. Fundo, M. Kreku, Mixture of a new integral transform and homotopy perturbation method for solving nonlinear partial differential equations, Advances in Pure Mathematics, 3, No. 3 (2013), 317-323.

15) Erd´elyi, A., Magnus, W., Oberhettinger, F. and Tricomi, F. G. (1953). Higher Transcendental Functions, Vol. 1, McGraw - Hill, New York, Toronto and London.

16) Erd´elyi, A., Magnus, W., Oberhettinger, F. and Tricomi, F. G. (1954). Tables of Integral Transforms, Vol. 1, McGraw - Hill, New York, Toronto and London.

17) Fox, C. (1963). Integral transforms based upon fractional integration, Proc. Cambridge Philos. Soc., 59, 63-71.

18) S. Lin, C. Lu, Laplace transform for solving some families of fractional differential equations and its applications, Advances in Difference Equations (2013), 1-9.

19) A. Kashuri, A. Fundo, Variational iteration method for fractional calculus by new integral transform, In: Conference: NTAM 2014, 7-8 November Tirana, Albania.

20) A. Kashuri, A. Fundo, R. Liko, new integral transform for solving some fractional differential equations, International Journal of Pure and Applied Mathematics Volume 103 No. 4 2015, 675-682

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